LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

M.Sc. DEGREE EXAMINATION - **MATHEMATICS**

SECOND SEMESTER – **NOVEMBER 2013**

MT 2962 - ACTUARIAL MATHEMATICS

Date : 18/11/2013 Dept. No. Max. : 100 Marks
Time : 1:00 - 4:00

Answer all Questions. All questions carry equal marks.

1. (a) Define distribution and survival functions of the time-until-death random variable $T(x)$ and obtain its expressions in terms of $S(x)$.

(OR)

(b) Prove that ${}_{t/u}q_x = {}_u q_{x+t}\,{}_t p_x = {}_{t+u}q_x - {}_t q_x$. (5 marks)

(c) (i) If $S(x) = \left[1 - \left(\frac{x}{100}\right)\right]^{\frac{1}{2}}, 0 \le x \le 100$, evaluate ${}_{17}p_{19}$, ${}_{15}q_{36}$ and ${}_{15/13}q_{36}$.

(ii) Verify whether the function $S(x) = e^{\frac{-x^3}{12}}, x \ge 0$, is a survival function. Find the corresponding $\mu(x)$, $f_X(x)$ and $F_X(x)$.

(iii) If $\mu(x) = 0.001, 20 \le x \le 25$, evaluate ${}_{2/2}q_{20}$. (5+6+4 marks)

(OR)

(d) (i)) For a given $F(x) = 1 - \frac{1}{x+1}, x \ge 0$, which of the following are true?

1) ${}_x p_0 = \frac{1}{x+1}$ 2) $\mu_{49} = 0.02$ and 3) ${}_{10}p_{39} = 0.81$ (6 marks)

(ii) Define curtate future life-time random variable $K(x)$ and obtain its probability mass function.

Find the distribution of $K(x)$ when $S(x) = 1 - \frac{x^2}{100}, 0 \le x \le 10$. Also obtain its expectation e_4. (9 marks)

2. (a) Suppose a survival model is defined by the following values of p_x.

x	:	0	1	2	3	4
p_x	:	0.9	0.8	0.6	0.3	0

What are the corresponding values of $S(x)$ for x=0, 1, 2, 3, 4 and 5?

(OR)

(b) Explain uniform distribution of deaths and hence prove that $l_{x+t} = l_x - t\,d_x$.

(5 marks)

(c) (i) Given $q_{60} = 0.3$ and $q_{61} = 0.4$, find the probability that (60.5) will die between (60.5) and (61.5) under the assumption of uniformity of deaths in the unit interval.

(ii) Under the assumption of uniform distribution of deaths in a unit interval, find ${}_{1.5}p_{30.5}$ and $\mu_{30.5}$ in terms of l_{30}, l_{31} and l_{32}.

(iii) There are 1000 persons of age 40 of whom 32 are expected to die before age 41. How many will die between $40\frac{3}{8}$ and $40\frac{3}{4}$ under the assumption of uniformity of deaths?

(5+5+5 marks)

(OR)

(d) (i) A mortality table has a select period of three years. Find expressions in term of life table functions $l_{[x]+t}$ and l_y for $q_{[50]}$, ${}_2q_{[50]}$, ${}_{2/}q_{[50]}$ and ${}_{2/3}q_{[50]+1}$.

(ii) A select and ultimate table with a three year select period begins at selection age 20. Given $l_{26} = 90,000$, $q_{[20]} = \frac{1}{6}$, $5p_{[21]} = \frac{4}{5}$ and $3p_{[20]+1} = \frac{9}{10}3p_{[21]}$, find $l_{[20]}$.

(8+7 marks)

3. (a) Find the amount to which Rs. 1, 000/- will accumulate at 6% per annum convertible half yearly for 5 years. In how many years will a sum of money double itself at compound interest with effective rate $I = 0.05$?

(OR)

(b) If the probability density function of the future life time T is given by

$$g(t) = \begin{Bmatrix} \frac{1}{80}, & 0<t<80 \\ 0 & elsewhere \end{Bmatrix},$$ then calculate the net single premium at a force of interest δ.

(5 marks)

(c) (i) An alumni association has 50 members, each of age x. It is assumed that all lives are independent. It is decided to contribute Rs. R to establish a fund to pay a death benefit of Rs . 10, 000/- to each member. Benefits are to be payable at the moment of death. It is given that $\overline{A_x} = 0.06$ and ${}^2\overline{A_x} = 0.01$. Using normal approximation, find R so that with probability 0.95 the fund will be sufficient to pay the death benefit.

(ii) Give an account of endowment insurance policy. **(OR)**

(d) (i) Assume that each of 100 independent lives is of age x, is subject to a constant force of mortality μ = 0.04 and is insured for a death benefit amount of 10 units, payable at the moment of death. The benefit payments are to be withdrawn from an investment fund earning interest at a rate δ = 0.06. Calculate the minimum amount to be collected at *t*=0, so that the probability is approximately 0.95 that sufficient funds will be on hand to withdraw the benefit payment at the death of each individual.

(ii) Give an account of whole life insurance policy. (9+6 marks)

4. (a) Define annuities and explain different classifications of annuities.

(OR)

b) Prove the following:

1) $a_{\overline{n|}} = \dfrac{1-v^n}{i}$, 2) $S_{\overline{n|}} = \dfrac{(1+i)^n - 1}{i}$.. (5 marks)

(c) (i) A loan of Rs. 50,000/- is taken on January1, 2000. It has to be repaid by 15 equal installments payable yearly at the beginning of the year. Based on an 8% annual rate of interest, determine the amount of installment.

(ii) Find the present and accumulated values of a 10-year annuity immediate of Rs. 1,000/- per annum if the effective rate of interest is 5%.

(iii) Prove that $\ddot{a}_x = \dfrac{1-A_x}{d}$ and $Var(\ddot{a}_{\overline{k+1|}}) = \dfrac{{}^2A_x - (A_x)^2}{d^2}$. (5+5+5 marks)

(OR)

(d) (i) For a 3-year temporary life annuity-due on (30), given $S(x) = 1 - \dfrac{x}{80}, 0 \le x < 80$, $i = 0.05$ and $Y = \begin{cases} \ddot{a}_{\overline{k+1}}, k = 0,1,2 \\ \ddot{a}_{\overline{3}}, \quad k=3,4,5 \end{cases}$, calculate *Var*(*Y*).

(ii) Given ${}_{10}E_x = 0.40$, ${}_{10/}\ddot{a}_x = 7$, $\ddot{S}_{x:\overline{10}/} = 15$ and $v = 0.94$, calculate $\ddot{a}_x$ and $A^1_{x:\overline{10|}}$. (8+7 marks)

5. (a) Define benefit premiums. A fully continuous 10-year term insurance of face amount Rs. 10,000/- has annual premium rate Rs. 100/- and the force of interest is 0.05. Find the value of the issue-date-loss if the death occurs exactly 5 years after issue.

(OR)

(b) Define loss function. On May 6, 1996, (67) bought a Rs. 1,00,000/- whole life insurance policy with death benefit payable at the end of the year of death. The policy is paid for by means of annual premiums, payable at the start of each year the policy remains in force. The policy holder died on August 6, 2003 and the loss to the insurer was Rs. 30,000/-. If *i*=0.06, what was the annual premium paid? (5 marks)

(c) (i) Prove that $\overline{P}(\overline{A}_x) = \dfrac{\overline{A}_x}{\overline{\overline{a}}_x}$ and $Var\ (L) = \dfrac{{}^2\overline{A}_x - (\overline{A}_x)^2}{(\delta \overline{a}_x)^2}$.

(ii) For a whole life insurance with unit benefit, calculate $\bar{P}(\bar{A}_x)$ and $Var\,(L)$ with the assumptions that the force of mortality is constant μ=0.04 and the force of interest δ=0.06.

(9+6 marks)

(OR)

(d) (i) L is the loss-at-issue random variable for a fully discrete whole life insurance of 1 on (49). Calculate P and E(L) where $A_{49} = 0.23882$, $\ddot{a}_{49} = 13.4475$, ${}^{2}A_{49} = 0.088873$, $i = 0.06$ and $Var(L) = 0.10$.

(ii) If ${}_{k|}q_x = c\left(0.96\right)^{k+1}, k = 0,1,2,...$ where c=0.04/0.96 and i=0.06, calculate P_x and Var(L).

(iii) For (x), given the following informations:

1) The premium for a 20-year endowment insurance of 1 is 0.0349.
2) The premium for a 20-year pure-endowment of 1 is 0.0230.
3) The premium for a 20-year deferred whole life annuity-due of 1 per year is 0.2087 and is paid for 20 years.
4) All premiums are fully discrete annual benefit premiums.
5) i=0.05.

Calculate the premium for a 20-payment whole life insurance of 1.

(6+4+5 marks)